32.21 **$50°F$ chilled water flows through a nominal $4in$ steel pipe wrapped with $2in$ thick insulation with thermal conductivity $0.05\frac{Btu}{hr\cdot ft\cdot °F}$. The ambient space is maintained at $74°F$ and the convection heat transfer coefficient between the insulation and the air is $2\frac{Btu}{hr\cdot ft^2\cdot °F}$. What is the surface temperature of the outside of the insulation?**

A. $66°F$

B. $68°F$

C. $70°F$

D. $72°F$

Draw a cross section of the insulated pipe. Use the table Schedule 40 Steel Pipe to look up the inside diameter and thickness for a nominal $4in$ pipe and calculate the outside diameter.

$$D_i = 4.026in$$

$$D_o = D_i + 2t = 4.026in + 2\left(0.237in\right) = 4.5in$$

Adding on the thickness of the insulation, the overall diameter of the *insulated* pipe is $4.5in + 2\left(2in\right) = 8.5in$. Therefore, the inner and outer radii for the insulation are $r_i = 2.25in$ and $r_o = 4.25in$, respectively.

Assume there is no conduction resistance through the pipe wall, and all conduction resistance is provided by the insulation. Determine the Cylindrical Wall Conduction Resistance. For convenience, select a $1ft$ section of pipe for the analysis.

$$R_{cond} = \frac{ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$$

$$R_{cond} = \frac{ln\left(\frac{4.25in}{2.25in}\right)}{2\pi\left(0.05\frac{Btu}{hr\cdot ft\cdot °F}\right)\left(1ft\right)} = 2.02\frac{hr\cdot °F}{Btu}$$

Determine the Convection Resistance from the outside of the insulation to the ambient space. The area is the surface area of the outside of the insulation, given by $A_s = \pi DL$. Again, select a $1ft$ section of pipe for the analysis.

$$R_{conv} = \frac{1}{hA}$$

$$R_{conv} = \frac{1}{\left(2\frac{Btu}{hr\cdot ft^2\cdot °F}\right)\left[\pi\left(\frac{8.5}{12}ft\right)\left(1ft\right)\right]} = 0.22\frac{hr\cdot °F}{Btu}$$

For the temperatures to be in equilibrium at steady state, the heat flux from the water through the insulation by conduction is equal to the heat flux from the outer surface of the insulation to the room by convection. Set these two quantities equal.

$$\dot{q}_{cond} = \dot{q}_{conv}$$

Express each side as the quotient of ΔT and resistance. Note the absence of area since these terms are already heat *flux* $\left[\frac{Btu}{hr \cdot ft^2}\right]$, not total heat transfer $\left[\frac{Btu}{hr}\right]$. The only unknown is the surface temperature of the insulation, T_s. Substitute known values and solve algebraically.

$$\frac{T_s - T_w}{R_{cond}} = \frac{T_\infty - T_s}{R_{cond}}$$

$$\frac{T_s - 50°F}{2.02 \frac{hr \cdot °F}{Btu}} = \frac{74°F - T_s}{0.22 \frac{hr \cdot °F}{Btu}}$$

$$T_s = 72.2°F$$

Answer D